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MATHEMATICS
STANDARD LEVEL
PAPER 2

Candidate session number

0	0							

Friday 10 May 2013 (morning)

Examination code

2	2	1	3	–	7	3	0	6

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[90 marks]*.

11 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 5]*

Let $\boldsymbol{A} = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ -1 & 1 & 1 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$.

(a) Write down $\boldsymbol{A}^{-1}$. *[2 marks]*

(b) Solve $\boldsymbol{AX} = \boldsymbol{B}$. *[3 marks]*

2. *[Maximum mark: 6]*

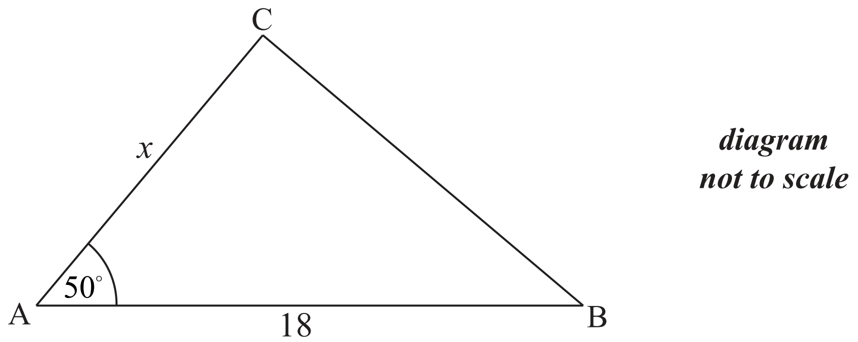
The random variable X is normally distributed with mean 20 and standard deviation 5.

(a) Find $P(X \leq 22.9)$. *[3 marks]*

(b) Given that $P(X < k) = 0.55$, find the value of k. *[3 marks]*

3. *[Maximum mark: 6]*

The following diagram shows a triangle ABC.



The area of triangle ABC is 80 cm^2, $\text{AB} = 18\text{ cm}$, $\text{AC} = x\text{ cm}$ and $\text{B}\hat{\text{A}}\text{C} = 50^\circ$.

(a) Find x. *[3 marks]*

(b) Find BC. *[3 marks]*

4. *[Maximum mark: 7]*

Line L_1 has equation $\boldsymbol{r}_1 = \begin{pmatrix} 10 \\ 6 \\ -1 \end{pmatrix} + s\begin{pmatrix} 2 \\ -5 \\ -2 \end{pmatrix}$ and line L_2 has equation $\boldsymbol{r}_2 = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} + t\begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix}$.

Lines L_1 and L_2 intersect at point A. Find the coordinates of A.

5. *[Maximum mark: 6]*

The sum of the first three terms of a geometric sequence is 62.755, and the sum of the infinite sequence is 440. Find the common ratio.

6. *[Maximum mark: 7]*

The constant term in the expansion of $\left(\frac{x}{a}+\frac{a^2}{x}\right)^6$, where $a \in \mathbb{Z}$, is 1280. Find a.

7. *[Maximum mark: 8]*

The following diagram shows a circle with centre O and radius r cm.

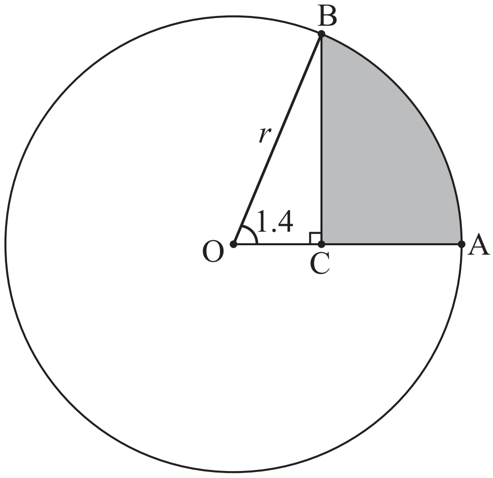


diagram not to scale

Points A and B are on the circumference of the circle and $A\hat{O}B = 1.4$ radians. The point C is on [OA] such that $B\hat{C}O = \frac{\pi}{2}$ radians.

(a) Show that $OC = r\cos 1.4$. *[1 mark]*

(b) The area of the shaded region is 25 cm^2. Find the value of r. *[7 marks]*

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

8. *[Maximum mark: 15]*

Consider the points $A(5, 2, 1)$, $B(6, 5, 3)$, and $C(7, 6, a+1)$, where $a \in \mathbb{R}$.

(a) Find

(i) $\overrightarrow{AB}$;

(ii) $\overrightarrow{AC}$. *[3 marks]*

Let α be the angle between $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

(b) Find the value of a for which $\theta = \frac{\pi}{2}$. *[4 marks]*

(c) (i) Show that $\cos\theta = \frac{2a+14}{\sqrt{14a^2+280}}$.

(ii) Hence, find the value of a for which $\alpha = 1.2$. *[8 marks]*

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 15]*

A bag contains four gold balls and six silver balls.

(a) Two balls are drawn at random from the bag, with replacement. Let X be the number of gold balls drawn from the bag.

(i) Find $\mathrm{P}(X = 0)$.

(ii) Find $\mathrm{P}(X = 1)$.

(iii) Hence, find $\mathrm{E}(X)$. *[8 marks]*

Fourteen balls are drawn from the bag, with replacement.

(b) Find the probability that exactly five of the balls are gold. *[2 marks]*

(c) Find the probability that at most five of the balls are gold. *[2 marks]*

(d) Given that at most five of the balls are gold, find the probability that exactly five of the balls are gold. Give the answer correct to two decimal places. *[3 marks]*

*Do **NOT** write solutions on this page.*

10. *[Maximum mark: 15]*

Let $f(x) = \mathrm{e}^{\frac{x}{4}}$ and $g(x) = mx$, where $m \geq 0$, and $-5 \leq x \leq 5$. Let R be the region enclosed by the y-axis, the graph of f, and the graph of g.

(a) Let $m = 1$.

(i) Sketch the graphs of f and g on the same axes.

(ii) Find the area of R. *[7 marks]*

(b) Consider all values of m such that the graphs of f and g intersect. Find the value of m that gives the greatest value for the area of R. *[8 marks]*

Please **do not** write on this page.

Answers written on this page
will not be marked.